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The Bias in the Conventional Test of the Expectations Theory: Resolving the Anomalies at the Very Short End of the Term Structure

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Abstract

While the expectations theory of the term structure of interest rates plays an important role in economics and finance, it has received relatively little empirical support. The most frequently used test, which I call the conventional test, has provided some hope for the expectations theory. When the conventional test is used, the slope of the yield curve predicts the correct direction of long-term changes in the short-term rate and explains a significant proportion of such changes. The failure of the conventional test to accept the expectations hypothesis is usually attributed to the potential for bias from several sources. Rather than explaining the failure of the conventional test; however, this paper accounts for it relative success. Motivated by research indicating that the conventional test yields results that are more favorable to the expectations hypothesis when it is less likely to hold and less favorable when it is more likely to hold, I show that the conventional test is generally biased in favor of the expectations theory. Indeed, I show that this bias led Hardouvelis (1988), Simon (1990) and Roberds, Runkle and Whiteman (1996) to accept the expectations theory in circumstances when it is unlikely to hold. Monte Carlo experiments confirm the low power and bias of the conventional test of the expectations theory.

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> *"The forecasting of short term interest rates by long term interest is, in general, so bad that the student may well begin to wonder whether, in fact, there really is any attempt to forecast." — Macauly (1938), p. 33.*

The expectations theory plays an important role in economics and finance, particularly in analyses of monetary policy, where short-term rates are believed to be determined by the market's expectation for the federal funds rate, which the Fed controls. Empirical support for the expectations theory is scant, however [e.g., Campbell and Shiller (1991) and Froot (1989)].

The two most commonly used tests of the expectations theory involve linear regressions of two different variables on the spread between the long-term rate and the short-term rate. In one test the dependent variable is the short-term change in the long-term rate. In the other, which I call the conventional test, the dependent variable is the long-term change in the short-term rate. While the expectations theory is generally rejected using either test, it fairs better when the conventional test is used. The rate spread nearly always indicates the correct direction for long-term changes in the short-term rate and the rate spread explains a significant proportion of such changes. The relative success of the conventional test has provided hope for the expectations theory. Indeed, the central question has been: why does the conventional test reject the expectations theory? While several explanations have been advanced [e.g., Fama (1984) and Mankiw and Miron (1986), McCallum (1994), Campbell and Shiller (1991) and Balduzzi, Bertola and Foresi (1997)], none has accounted for the expectations theory's failure [e.g., Balduzzi, Bertola and Foresi (1997), Hsu and Kugler (1997), Hardouvelis (1994), Simon (1990) and Froot (1989)].[1]

This paper takes a fundamentally different approach. Rather than explaining the failure of the conventional test, this paper explains its relative success. Specifically, I show that the conventional test is biased in favor of accepting the expectations theory when it is false, i.e., the conventional test lacks power. This paper is motivated by Hardouvelis (1988), Simon (1990) and Roberds, Runkle and Whiteman's (1996) result that the conventional test accepts the expectations theory in circumstances where, *a priori*, the market should have a relatively difficult time predicting the funds rate and rejects it when the funds rate should have been relatively easy to predict.

The outline of the paper is as follows. Section I presents the conventional test of the expectations theory and shows that the test is biased toward accepting the expectations theory if it is false. Section II explains why the results obtained by Hardouvelis (1988), Simon (1990) and Roberds, Runkle and Whiteman (1996) are anomalous. Since these authors appeal to a hypothesis advanced by Mankiw and Miron (1986) to rationalize their findings, this section also shows why Mankiw and Miron's hypothesis cannot account for their results. Section III presents the empirical results and demonstrates how the bias in the conventional test led these researchers to accept the expectations theory in unlikely circumstances. In Section IV Monte Carlo mehtods are used to investigate the size and power of the conventional test.

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[1] Tzavalis and Wickens (1997) claim to account for the failure of both tests. Specifically, they add an autoregressive term premium that they argue accounts for nearly all of the bias. It should be noted, however, that if one accounts for all of the variation in the rate spread with a time-varying term premium, the expectations theory is effectively a tautology. Tzavalis and Wickens' estimates of $\overline{R}^2$ are nearly 1 in all of their equations, suggesting that they accept the expectations theory solely because they account for all of the residual variation with their time-varying term premium.

I. The Conventional Test of the Expectations Theory

The expectations theory of the term structure is a hypothesis about the relationship between a long-term, *n*-period interest rate, R_t^n, and expected future levels of a short-term, *m*-period rate, *n*-*m* = (*k*-1)*m* periods in the future, where *k* = *n*/*m* is an integer. That is,

$$(1) \qquad R_t^n = (1/k)\sum_{i=0}^{k-1} E_t R_{t+mi}^m + \pi .$$

Equation 1 states that the *n*-period rate is equal to the average of the market's expectation for the *m*-period rate over the term of the *n*-period rate plus a constant risk premium, π.[2]

The conventional test of the expectations theory is derived by assuming that expectations are rational, i.e.,

$$(2) \qquad E_t R_{t+mi}^m = R_{t+mi}^m + \nu_{t+mi}, \quad i = 0,1,\ldots,k-1,$$

where ν_{t+mi} is a mean zero, iid white noise error. Equation 2 is then substituted into Equation 1, which yields,

$$(3) \qquad R_t^n = (1/k)\sum_{i=0}^{k-1} R_{t+mi}^m + (1/k)\sum_{i=0}^{k-1} \nu_{t+mi} + \pi .$$

The next step in the derivation is critical. Specifically, the current *m*-period rate is subtracted from both sides of Equation 3 in order to express Equation 3 in terms of the rate spread, i.e.,

$$(4) \qquad (1/k)\sum_{i=0}^{k-1} R_{t+mi}^m - R_t^m = -\pi + (R_t^n - R_t^m) - \omega_t$$ [3]

The conventional test of the expectations theory is obtained by *parameterizing* Equation 4 to yield,

$$(5) \qquad (1/k)\sum_{i=0}^{k-1} R_{t+mi}^m - R_t^m = \alpha + \beta(R_t^n - R_t^m) + \varpi_t .$$

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[2] Shiller, Campbell and Schoenholtz (1983) argue that Equation 1 is exact in some special cases and that it can be derived as a linear approximation to a number of nonlinear expectations theories of the term structure.

[3] Here, $\omega_t = (1/k)\sum_{i=0}^{k-1} \nu_{t+mi}$.

If the expectations theory is true, the null hypothesis that β = 1 should not be rejected.

Subtracting the *m*-period rate might seem innocuous, but it introduces the possibility of spurious regression bias.[4] It is fairly obvious from the structure of Equation 5 that even if the *n*-period and *m*-period rates were independent, the estimate of β would be positive if the *m*-period rate is more variable than the *n*-period. This observation can be obtained more formally by parameterizing Equation 3 before the short-term rate is subtracted from both sides. That is, rewrite Equation 3 as

$$(6)\qquad (1/k)\sum_{i=0}^{k-1} R_{t+mi}^{m} = \beta R_t^n - (1/k)\sum_{i=0}^{k-1} v_{t+mi} - \pi.$$

Subtracting the short-term rate from both sides of Equation 6, as before, yields,

$$(7)\qquad (1/k)\sum_{i=0}^{k-1}\ R_{t+mi}^{m} - R_t^m = \alpha + \beta(R_t^n - R_t^m) + (\beta - 1)R_t^m + \varpi_t\,.$$

Note that Equation 7 reduces to Equation 5 if and only if $\beta = 1$, i.e., if only if the expectations theory is true. If the expectations theory is not true, however, the expected value of the least squares estimator of β from Equation 5, $\hat{\beta}$, is equal to

$$(8)\qquad E\hat{\beta} = \beta + (\beta - 1)E\,\frac{\sum(\overline{R}_t^n - \overline{R}_t^m)\overline{R}_t^m}{\sum(\overline{R}_t^n - \overline{R}_t^m)^2},$$

where the bar denotes that the variable has been adjusted for the mean. The second term on the right hand side of Equation 8 is zero only when the expectations theory holds, i.e. β = 1. When $\beta \neq 1$, however, the esimate of β will be biased. Moreover, the bias will not disappear in large samples, i.e.,

$$(9)\qquad \underset{N\to\infty}{P\lim}\,\hat{\beta} = \beta + (\beta - 1)\left[\frac{\sigma_{nm} - \sigma_m^2}{\sigma_n^2 - 2\sigma_{nm} + \sigma_m^2}\right].$$

[4] Concern about spurious correlation in such instances dates back to Pearson (1897); see Kuh and Meyer (1955).

Noting that $\sigma_{nm} / \sigma_n^2 = \rho\delta$, this expression can be rewritten as

$$(10) \qquad \underset{N\to\infty}{P\lim}\hat{\beta} = \beta + (\beta - 1)\left[\frac{\rho\delta^{1/2} - \delta}{1 - 2\rho\delta^{1/2} + \delta}\right],$$

where ρ is the coefficient of correlation between R_t^m and R_t^n and $\delta = \sigma_m^2 / \sigma_n^2$. The bias depends on both the extent to which β differs from 1 and the term in brackets, which I call the *bias factor*, *bf*. In general *bf* can either be positive or negative; however, it is strictly negative when $\delta > 1$—a prediction of the expectations theory. Hence, in all cases relevant for the expectations theory, $\hat{\beta}$ will be biased in favor of it.

In the special case where the *n*-period and *m*-period rates are independent, i.e., $\rho = 0$, $bf = -\delta / (1 + \delta)$. Hence, $\hat{\beta}$ and *bf* approach 1 and –1, respectively, as $\delta \to \infty$. When $\rho > 0$, as is always the case with interest rates, $bf = -1$ for all values of δ and ρ, such that $\delta = \rho^{-2}$. Hence, it is possible for $\hat{\beta}$ to be biased toward 1 when the variance of the *m*-period rate increases even modestly but independently of that of the *n*-period rate.

This is illustrated in Figure 1, which shows *bf* for $\rho \in (0.01, 0.99)$ and $\delta \in (0.01, 10)$ for all combinations of δ and ρ such that $\delta = \rho^{-2}$, $bf = -1$. Figure 1 makes it clear while the conventional test will be biased in favor of accepting the expectations theory in situations where the volatility of the short-term rate increases relative to that of the long-term rate. When the variance of the short-term rate increases, not only does δ increase but, perhaps more importantly, ρ falls. Figure 1 shows why that the combination of these two effects can significantly move the estimate of β toward 1. Hence, in the case where the short-term and long-term rates are highly correlated, even a relatively small increase in the variability of the short-term rate can significantly bias the estimate of β to 1. Of course, the bias depends on the size of both *bf* and the true value of β. In cases where $bf < -1$, the bias can be even greater than 1. This fact might explain why in some cases researchers have produced estimates of $\hat{\beta}$ significantly larger than 1.[5]

II. Anomalies at the Very Short End of the Term Structure

Hardouvelis (1988), Simon (1990) and Roberds, Runkle and Whiteman (1996) test the expectations theory using Equation 5 with the effective federal funds rate as the short-term rate and the 3-month T-bill rate as the long-term rate. Their results are reminiscent of those in the literature. Estimates of β are positive and estimates of $\bar{R}^2$ are significantly larger than zero; however, with two exceptions, the null hypothesis that $\beta = 1$ is rejected. The first exception is when the test is performed for the period of nonborrowed reserves targeting, October 1979-October 1982. The second is when Equation 5 is estimated only using reserve market settlement days, called settlement Wednesdays. When settlement Wednesdays are used the estimates of β are much closer to 1 and in a few cases the hypothesis that $\beta = 1$ is not rejected.

These exceptions appear to be anomalies. The expectations theory depends critically on the market's ability to forecast the level of the short-term interest rate, in this case, the federal funds rate. Generally speaking, the better the market forecasts the federal funds, the better the expectations theory should perform. If the Fed conducts monetary policy rationally and if the market forms rational expectations of Fed policy, *a priori*, the expectations theory should perform relatively well when the Fed is targeting the funds rate in a narrow band and relatively poorly when it is not. During the 1979-82 period the Fed was targeting M1 growth using a nonborrowed reserves operating procedure. During periods before and after this one, the Fed

[5] For example, Campbell and Shiller (1991) report that the estimate of β from Equation 5 is 2.788 when the maturity of the long-term rate is ten years and the maturity of the short-term rate is 5 years. The 5-year and 10-year rates are highly correlated and the 5-year rate is only slightly more variable than the 10-year rate. Indeed, Campbell

was targeting the federal funds rate. *A priori*, the market should have more difficulty forecasting the funds rate when the Fed is targeting the money stock and less difficulty when the Fed is targeting the funds rate, but the results from the conventional test indicate that just the opposite is true. Likewise, it is difficult to see how the sometimes large and transient movements in the federal funds rate, which tend to occur on settlement Wednesdays, could improve the market's forecast of the funds rate 90 or 181 days into the future.

A. The Mankiw and Miron Hypothesis

Using the conventional test Mankiw and Miron (1986) found that the slope of the yield curve had predictive power for the path of short-term rates before the founding of the Fed but not after. They attributed the failure of the expectations theory after the Fed's founding to the Fed's practice of smoothing short-term interest rates. Mankiw and Miron (1986), pp. 225-26, state,

> "During much of this period the Fed's announced policy was to stabilize (or even to peg) interest rates. One simple description of interest rate stabilization is…
>
> (10) $$\Delta r_{t+1} = 0;$$
>
> that is, the change in the short rate is zero. The data, however, obviously reject this characterization of policy, since the short rate did change throughout this period. A second, less restrictive description of Fed policy is
>
> (11) $$E_t \Delta r_{t+1} = 0;$$
>
> that is, the *expected* change in the short rate is zero.…If equation (11) does describe the policy of stabilizing interest rates and if market participants knew it was the policy, then the short rate expected by the market would always equal the current short rate. The spread ($R_t - r_t$) would always equal the term premium θ_t. Fluctuations in the spread would have no predictive power for the path of the short rate. Thus, the failure of the expectations theory with data from the 1960s and 1970s, a fact documented here and in many previous studies, may be an ineluctable result of Federal Reserve policy during this period."

Mankiw and Miron's hypothesis is important because Roberds, Runkle and Whiteman (1996) use it to rationalize their settlement Wednesday results and Hardouvelis (1988) and

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and Shiller's (1991) estimates of β are always greater than 1 when the maturity of the long-term rate is five years or

Simon (1990) argue that it explains why the expectations theory is rejected when the Fed is targeting the federal funds rate. I show why it is unlikely that Mankiw and Miron's indictment of the conventional test of the expectations theory—in the extreme circumstances they describe—accounts for either of these results.

B. The Mankiw and Miron's Hypothesis and the Settlement Wednesday Results

Roberds, Runkle and Whiteman (1996) argue that the settlement Wednesday result is due to the fact that large changes in the federal funds rate, that frequently occur on settlement Wednesdays, are offset the following day. Specifically, Roberds, Runkle and Whiteman (1996), pp. 49-50 state,

> "Our settlement-day results might be considered noteworthy in the sense that they show that the markets are not "spooked" by settlement-day pressures in the overnight Fed funds market. The market believes that the Fed is committed to returning to the presettlement overnight funds rates after settlement Wednesday, no matter how much rates move on settlement Wednesday. Thus yield spreads on settlement Wednesday are good predictors of future movements in short rates."

Rudebusch (1995), p. 269, who found that transitory deviations in the funds rate accounted for much of the spread's explanatory power in his simulations, clarifies Roberds, Runkle and Whiteman's (1996) rationale, stating,[6]

> "…if today's spot rate is unusually high relative to the target, it can be expected that future daily rates will return to the target level. Thus, the current three-month rate is close to the target rate. In this way, the spread between the funds rate and the three-month rate should be a very good predictor of the *change* from the current daily rate to the average rate daily rate that prevails over the next three months."

The idea is that when the funds rate deviates substantially from the target level, the market has firm expectations of the funds rate's behavior the next day. Specifically, $E_t \Delta i_{t+1}^m = -\Delta i_t^m$. To see

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more.

[6] Campbell, Lo and MacKinlay (1997), p. 423 also accept this explanation.

the implication of this argument for the expectations theory, assume that the Fed targets the funds rate and keeps it close to the funds rate target, i.e.,

(11) $ff_t = ff_t^* + \nu_t$,

where ff_t^* denotes the Fed's funds rate target on day t and $\nu_t \sim iid(0, \sigma_\nu^2)$ denotes the Fed's control error. The expectations theory hypothesizes that the 91-day T-bill rate, tb, is determined by,

(12) $tb_t = (1/91)\sum_{i=0}^{90} E_t ff_{t+i} + \pi$.

Substituting Equation 11 into 12 yields,

(13) $tb_t = (1/91)\sum_{i=0}^{90} E_t (ff_{t+i}^* + \nu_{t+i}) + \pi$.

Using Mankiw and Miron's explicit assumption, which is implicit in Rudebusch's (1995) explanation, that the funds rate is observed at time *t*, Equation 13 can be rewritten as,

(14) $tb_t = (1/91)\sum_{i=0}^{90} ff_{t+i}^* + \nu_t / 91 + \pi$.[7]

Equation 14 shows that knowledge of the funds rate on day *t* improves the markets ability to forecast the average funds rate over the holding period; however, the improvement is relatively modest and declines with the term of the T-bill.[8]

This example makes it clear that an essential element of Rudebusch's (1995) argument is that the market knows the funds rate when the T-bill rate is determined. Otherwise, $E_t \nu_t = 0$ and there is absolutely no reason to expect the expectations theory to work better—even marginally better—on reserve settlement days. This assumption cannot hold in this case because the effective funds rate is not reported until the next day. Even if the funds rate were observed on t,

[7] Mankiw and Miron (1986), p. 213.
[8] Indeed, knowledge of the funds rate on day t would be particularly useful only for small values of k.

however, it could not account for the difference in the results on reserve on settlement days and other days because the market would have the same informational advantage every day.

What can account for the differential results is the increased variability of the funds rate. Large settlement-day increases in the funds rate cause the left- and right-hand-sides of Equation 5 to be large and negative. If the funds rate is unusually soft on settlement Wednesdays, the left- and right-hand-sides of Equation 5 will be large and positive. The increase in the variability of the funds rate on settlement Wednesdays relative to other days could bias the estimate of β to 1. We will see that it is this feature of settlement Wednesdays that accounts for Roberds, Runkle and Whiteman's (1996) settlement day results.

C. The Mankiw and Miron's Hypothesis and the 1979-82 Results

Hardouvelis (1988) and Simon (1990) attribute their finding that the expectations theory is rejected except when the Fed is targeting M1 to the Fed's practice of interest rate smoothing. This explanation seems unlikely, however, because the expectations theory is a hypothesis about the predictability of the level of the short-term interest rate. *A priori*, the federal funds rate should be more predictable when the Fed is targeting it than when it is permitted to fluctuate as the Fed attempts to offset the effects of demand shocks on the money supply.

Of course, the conventional test would perform poorly as Mankiw and Miron suggest, if the Fed maintained the funds rate target constant for long periods of time, but this has not been the case. Indeed, Rudebusch (1995) reports that the Fed made 98 adjustments to its funds rate target during the period September 13, 1974 to September 19, 1979, an average of a target change about every 2.5 weeks. Since 1983 the funds rate target has been changed 122 times—more than once every two months.

Hence, Mankiw and Miron's hypothesis can only account for the failure of the expectations theory during the periods of funds rate targeting if market participants believed that each target change was the Fed's last.[9] That is, the market would have to form irrational expectations of Fed policy.[10]

Aware that the expectations theory should work better when the level of the short rate is more predictable, Hardouvelis (1988) and Simon (1990) investigate the relationship between their results and the general predictability of the federal funds rate.[11] They found that the predictability of the funds rate did not vary significantly with the monetary policy operating procedure. Despite these findings, both authors concluded that their results were consistent with the Mankiw and Miron's hypothesis.

III. Data and Estimates

The data used to test the expectations theory are the effective federal funds rate, *ff*, and the 3-month T-bill rate, *tb*. The effective federal funds rate is a weighted average of all daily transactions for a group of brokers who report daily to the Federal Reserve Bank of New York. The observations are daily over the period September 23, 1974 to December 31, 1997. Equation 5 is estimated over the entire period and over sub-periods corresponding to different operating objectives of the Fed. The first is the period of funds rate targeting, September 23, 1974 to October 5, 1979. The second is the nonborrowed reserves targeting period, October 9, 1979 to October 6, 1982. The third is the period of indirect or *fuzzy* [Goodfriend (1991)] funds rate targeting, October 7, 1982 to August 10, 1987. During this period, the Fed maintained that it

[9] Actually, the opposite is true. Once the target is adjusted, the market typically assumes that more adjustments in the same direction are forthcoming because this has been the Fed's practice.

[10] A number of analysts [e.g., Goodfriend (1991), Poole (1991) and Rudebusch (1995)] argue that rational policy making means the Fed has an incentive to be transparent about its intentions for the funds rate.

[11] Hardouvelis (1988) used a univariate AR model of the funds rate and a VAR. Simon (1990) used the Goldsmith-Nagan *Reporting on Governments and Bonds and Money Market Letter* survey.

was targeting borrowed reserves; however, the evidence [Thornton (1988) and Feinman (1993)] suggests that the Fed was targeting the funds rate. The final period is a period of explicit funds rate targeting. These periods are similar to those used by Simon (1990) and Roberds, Runkle and Whiteman (1996).

Estimates of Equation 5, presented in Table 1, are very similar to those reported by Simon (1990) and Roberds, Runkle and Whiteman (1996).[12] The estimate of β for the entire period of 0.54 is very similar to Simon's estimate of 0.5, and the estimates over the various sub-periods correspond closely to those reported by Roberds, Runkle and Whiteman (1996). In particular, the estimate of β during the nonborrowed reserves targeting period is somewhat larger than, but not significantly different from, 1. The larger estimates of $\overline{R}^2$ during this period suggests that the spread explains significantly more of the variation in the future federal funds rate during this period. Moreover, while the expectations theory is rejected when only settlement Wednesdays are used, the estimate of β is closer to 1 and estimate of $\overline{R}^2$ is larger than for all days.

A. The Settlement Wednesday Results

The fact that estimates of β can be biased toward 1 when the variance of the short-term rate is high relative to the variance of the long-term rate, and the fact that Roberds, Runkle and Whiteman's (1996) explanation cannot account for the settlement Wednesday results, suggests

[12] The moving average of the federal funds rate on the left-hand-side of Equation 5 is calculated on a calendar-day basis. To do this, the last observation prior to a missing observation [i.e., a weekend or a holiday] was used to fill in missing observations. These observations were not used in the estimation, which used only market observations. Because the regression involves a moving average term, the residuals from this equation follow a moving average process of the same duration [e.g., Hansen and Hodrick (1980)]. Consequently, the estimated covariance is corrected for both serial correlation and heteroskedasticity using a procedure suggested by Hansen (1982). Because the number of market days is less than the number of calendar days used to calculate the moving average term, the maximum order of the process is set at 65, the maximum number of market days in a 3-month period. It was never the case that the estimated variance-covariance matrix was not positive semidefinite. Consequently, the Newey-West (1987) correction was not employed.

that the settlement-day results could be due solely to the increased variability of the funds rate on settlement Wednesdays. If this is the case, estimates of β should be closer to 1 whenever the variability of the funds rate increases.

To investigate this possibility, days when there are large shocks to the federal funds rate, called *large shock days*, LSDs, are identified. A LSD occurs when the federal funds rate changes by 80 basis points or more (approximately two standard deviations of the daily change over the sample period). Not surprisingly, of the 338 LSDs during the sample period, 120 were settlement Wednesdays.

Equation 5 was estimated over various subsets of the data over the entire sample. The results are presented in Table 2. Consistent with the analysis in Section 2, the estimate of β is closer to 1 and $\overline{R}^2$ is larger when Equation 5 is estimated using only LSDs. Moreover, the estimate of β gets smaller and estimates of $\overline{R}^2$ fall by nearly 40 percent when Equation 5 is estimated over the entire sample omitting LSDs. When the equation is estimated with LSDs that are settlement Wednesdays, the coefficient is only slightly larger than that for LSDs. The small difference in the estimates suggests that the important factor is whether the day is a LSD, not whether it is a settlement Wednesday. This is confirmed by the estimates in the last row of Table 2. When Equation 5 is estimated using settlements Wednesdays that are not LSDs, the estimate of β drops to 0.39 and only about 11 percent of the long-term change in the funds rate is explained by the slope of the yield curve. There is nothing unique about settlement Wednesdays other than the funds rate tends to be more variable when the aggregate reserve requirement for the banking system is binding.

The effect of LSDs—including settlement Wednesdays—on the estimate of β is illustrated in Figure 2, which shows $(1/91)\sum_{i=0}^{90} ff_{t+i} - ff_t$ and $tb_t - ff_t$, over the period September 23, 1974 to October 2, 1997. To emphasize their effect, LSDs are highlighted. A duel scale is used to make it easier to distinguish between the two series. It is clear that large changes in the funds rate produce nearly proportionate movements in the left- and right-hand-sides of Equation 5, biasing the estimate of β toward 1.

Finally, it is interesting to note that the above results are inconsistent with the usual time-varying term premium explanation of why the conventional test fails. A time-varying term premium means that the long-term rate changes relative to the market's expectation for the short-term rate. For the time-varying term premium explanation to be correct, the left-hand-side of Equation 5 would not change, while the right-hand-side changes because the long-term rate moves relative to the short-term rate. This is not what happens on settlement Wednesdays. On settlement Wednesdays or LSDs both the left- and right-hand-sides of Equation 5 move about equally.[13]

B. The Nonborrowed Reserves Targeting Period

While the effect of large shocks to the funds rate is the same during the period of nonborrowed reserves targeting, the estimate of β remains close to 1 and $\bar{R}^2$ remains relatively large even when LSDs are omitted. The reason for this is shown in Figure 3, which plots

[13] Moreover, Hardouvelis (1994) argues that the time-varying term premium explanation can account for the conventional test's failure to accept the expectations theory only if the term premium is highly volatile. It is easy to show that the bias due to a time-varying term premium is equal to $-\sigma_{\pi s} / \sigma_s^2$, where the numerator is the covariance between the term premium and the spread between the long-term rate and the short-term rate and the denominator is the variance of the rate spread. The bias can be rewritten as $-\rho(\sigma_\pi / \sigma_s) = -z$, where z is typically about .4 or larger. Unless the term premium and spread are very highly correlated, σ_π would have to be significantly larger than σ_s.

$(1/91)\sum_{i=0}^{90} ff_{t+i} - ff_t$, $tb_t - ff_t$, tb_t and ff_t over the 1979-82 period. Figure 3 shows that the federal funds rate tends to rise relative to the T-bill rate when interest rates are rising and fall relative to the T-bill rate when interest rates are falling. The increased variability of the funds rate over the interest rate cycle created a counter-cyclical pattern in $tb_t - ff_t$, i.e., $tb_t - ff_t$ tended to fall when rates were rising and tended to rise when rates were falling. The changes in the spread between the funds rate and the T-bill rate created a similar counter-cyclical pattern in $(1/91)\sum_{i=0}^{90} ff_{t+i} - ff_t$ because of the moving average term. When rates were rising, the moving average term, $(1/91)\sum_{i=0}^{90} ff_{t+i}$, rose more slowly than the funds rate, so that $(1/91)\sum_{i=0}^{90} ff_{t+i} - ff_t$ tended to fall when interest rates were rising. For analogous reasons $(1/91)\sum_{i=0}^{90} ff_{t+i} - ff_t$ tends to rise when interest rates are falling. The counter-cyclical behavior of $(1/91)\sum_{i=0}^{90} ff_{t+i} - ff_t$ is always present, but is particularly pronounced during this period because interest rates varied more dramatically over interest rate cycles during this period than in the others.

The estimate of β close to 1, and the larger estimates of $\overline{R}^2$, is due to the common cyclical pattern these variables caused by the funds rate moving relative to the T-bill rate, rather than the other way around. Hence, $tb_t - ff_t$ changed not because the T-bill rate changed, portending a change in the funds rate, but because the funds rate changed relative to the T-bill rate.

The federal funds rate averaged 200 basis points above the T-bill rate during this period, nearly four times the average spread over the rest of the sample period.[14] Exactly why the spread was so large during this period and why it tended to widen when interest rates rose and narrow when interest rates fell is unclear. Whatever the explanation for the increased variability of the funds rate, the failure to reject the null hypothesis $\beta = 1$ is due to this fact and not because the T-bill rate was anticipating movements in the funds rate.

IV. Monte Carlo Experiments

In this section, the power of the conventional test of the expectations theory is investigated using Monte Carlo methods. While the results in Section I show that the conventional test may lack power, estimates of β depend on the true data generating process, DGP. Indeed, Bekaert, Hodrick and Marshall (1997) show that if the if the short-term rate is generated by a simple AR(1) process [or a more general VAR-GARCH model] and the expectations hypothesis holds, estimates of β are significantly larger than 1, on average, even in fairly large samples. Based on their Monte Carlo evidence, they conclude that the results using the conventional test are less favorable to the expectations theory than is usually thought.

This section presents evidence from three Monte Carlo experiments. The first replicates Bekaert, Hodrick and Marshall (1997). In the second and third experiments, however, the path of the long-term rate is independent of the future short-term rate. Hence, the last two experiments investigate the power of the conventional test.

In the first experiment, the short-term rate is determined by the following AR(1) model,

(15) $$R_t^m = \mu^m + \psi R_{t-1}^m + \sigma_\gamma \gamma_t,$$

where $\gamma_t \sim N(0,1)$, and the long-term rate is determined by Equation 1.

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[14] The average spread over the remainder of the sample period was 52 basis points.

In the second experiment the long-term rate is determined by

(16) $$R_t^n = \mu^n + \varphi R_{t-1}^n + \sigma_\varepsilon \varepsilon_t,$$

and the short-term rate is determined by,

(17) $$R_t^m = R_t^n + \theta_t,$$

where $\phi(L)\theta_t = \mu^\theta + \sigma_\eta \eta_t$ and where ε_t and η_t are distributed $N(0,1)$ and are independent. The long-term rate is determined by an AR(1) process, while the short-term rate is equal to the long-term rate plus a random variable that exhibits a degree of persistence determined by $\phi(L)$. The long-term rate is determined by its own history, independent of the future path of the short-term rate.

In the third experiment the long-term and short-term rates are generated by independent AR(1) processes. Specifically,

(18) $$\begin{aligned} R_t^n &= \mu^n + \varphi R_{t-1}^n + \sigma_\varepsilon \varepsilon_t \\ R_t^m &= \mu^m + \psi R_{t-1}^m + \sigma_\gamma \gamma_t \end{aligned}.$$

Again, ε_t and γ_t are independent $N(0,1)$ stochastic processes.

All of these models are parameterized to be consistent with the time-series behavior of the observed federal funds and T-bill rates over the period October 7, 1982 to December 31, 1997. To conserve on space, the results are presented only for a sample of 450 observations.

The results are summarized in Table 3. Consistent with the finding of Bekaert, Hodrick and Marshall (1997), the mean estimate of β is greater than 1 in the first experiment. Hence, if the expectations theory is true and the funds rate is determined by an AR(1) process, estimates of β should be larger than those reported here and elsewhere.

Moreover, the conventional test of the expectations hypothesis is rather badly sized. The null hypothesis that $\beta = 1$ was rejected about 40 percent of the time at the 5 percent significance level. This occurs not solely because the estimates of β are less than one, but frequently because they are greater than one.

The results for the second experiment show that the conventional test lacks power. Despite the fact that the long-term rate was determined by its own history, the mean estimate of β was about .86. The null hypothesis that $\beta = 1$ was rejected only about forty percent of the time. About 60 percent of the time one would conclude that the expectations theory is true. Moreover, on average, one would conclude that the spread between the long-term rate and the short-term rate explains about 50 percent of the future behavior of the short-term rate.

The average estimate of β is in line with the average from rolling regression estimates of Equation 5 for a sample size of 450 over this same period. The rolling regression estimates of β range form 0.332 to 1.457, with a mean of 0.837—very close to the average reported in Table 3. While the average $\overline{R}^2$ of 0.376 from the rolling regressions is somewhat smaller than that reported in Table 3, the range of estimates, 0.065 to 0.856, is similar.

Some might argue that the Monte Carlo results mirror those of the actual data because the DGP produces data that is statistically similar to that produced by the true DGP. Hence, the third experiment assumes that the long-term and short-term rates are generated by independent AR(1) processes. Even in this extreme case, the null hypothesis that the expectations theory is true would be rejected only about 85 percent of the time at the 5 percent significance level. Moreover, the mean estimate of β is close to many of the estimates obtained using historical data, and the estimate of $\overline{R}^2$ suggests that, on average, the spread between the long-term rate and the short-term rate “explains” about fifty percent of the future movement in the short-term rate.

It is interesting to note that in the last two experiments, the probability of rejecting the hypothesis $\beta = 0$ was over 90 percent, at the 5 percent significance level. Thus, it is very likely that the conventional test will generate results supportive of the expectations theory even when it is false. These Monte Carlo results suggests that researchers should take little solace from the fact that the conventional test yields estimate of β that are positive and significantly different from zero or that the spread appears to "explain" a significant proportion of the future behavior of the short-term rate.

The positive mean estimate of β in the last two experiments is a direct consequence of the fact that the short-term rate appears on both sides of Equation 5. This is most easily demonstrated for the last experiment. To simplify the derivation, assume that the means of both processes are zero, so that Equation 5 can be written as,

$$(19) \quad \left[\frac{1-\psi^k}{k(1-\psi)} - 1\right] R_t^m + \Phi(\gamma_{t+h}) = \alpha + \beta(R_t^n - R_t^m) + \varpi_t, \quad h \geq 1,$$

where $\Phi(\gamma_{t+h})$ is a function involving γ dated $t+1$ or higher, so that $E\Phi(\gamma_{t+h})\gamma_t = 0$. Now note that Equation 19 can be rewritten as,

$$(20) \quad \left[\frac{1-\psi^k}{k(1-\psi)} - 1\right] \sum_{j=0}^{\infty} \psi^j \gamma_{t-j} + \Phi(\gamma_{t+h}) = \alpha + \beta\left(\sum_{j=0}^{\infty} \varphi^j \varepsilon_{t-j} - \sum_{j=0}^{\infty} \psi^j \gamma_{t-j}\right) + \varpi_t, \quad h \geq 1.$$

From Equation 20, it is easy to show that

$$(21) \quad \underset{N\to\infty}{P\lim}\, \hat{\beta} = \left[1 - \frac{1-\psi^k}{k(1-\psi)}\right](1+\delta^*)^{-1},$$

where, δ^* is the ratio of the variance of the long-term rate to the variance of the short-term rate, i.e., $\delta^* = \dfrac{\sigma_\varepsilon^2/(1-\varphi^2)}{\sigma_\gamma^2/(1-\psi^2)}$. Note that even when the long-term and short-term rates are generated by independent AR(1) processes, asymptotically, the estimates of β will be positive for $k > 1$.[15] Note too that the estimate of β will be larger the smaller is ψ. Moreover, consistent with the analysis of Section I, the estimate of β is larger the larger the variance of the short-term rate relative to the variance of the long-term rate, i.e., the smaller is δ^*.

V. Conclusions

Regardless of the test used, the expectations theory is nearly always rejected. Relative to other tests, however, a test, which I call the conventional test, tends to yield results that are generally supportive of the expectations hypothesis. Encouraged by the relative success of the expectations hypothesis using the conventional test, some effort has been devoted to account for the fact that, strictly speaking, the expectations theory is nearly always rejected when the conventional test is used, but to little avail.

Motivated by research that shows that the conventional test generates results that are more favorable to the expectations hypothesis when the federal funds rate should be less predictable and less favorable when the funds rate should be more predictable, this paper shows that the conventional test is biased in favor of the expectations theory when the expectations theory is false.

This bias is more severe in exactly those instances where Hardouvelis (1988), Simon (1990) and Roberds, Runkle and Whiteman (1996) find the most support for the expectations theory using the effective federal funds rate as the short-term rate. These authors appeal to a hypothesis advanced by Mankiw and Miron (1986) to explain their results. Hence, I show why it is unlikely that Mankiw and Miron's hypothesis accounts for their findings. I then present

[15] Note that $(1-\psi^k)/(1-\psi) = 1+\psi+\psi^2+\ldots+\psi^{k-1} < k$, for $|\psi| < 1$ and $k > 1$.

evidence that Hardouvelis (1988), Simon (1990) and Roberds, Runkle and Whiteman's (1996) findings are due to the bias in the conventional test.

The low power of the conventional test suggested by the analysis of these results is confirmed by Monte Carlo experiments. The Monte Carlo experiments show that no comfort can be taken from the relative success of the conventional test. Indeed, the slope of the yield curve has considerable "predictive power" for the long-term changes in the short-term rate when the long-term and short-term rates are generated by independent stochastic processes.

The present inquiry is limited to the relationship between the federal funds and T-bill rates; however, the results are likely to have broader application. Specifically, the bias in the conventional test might account for the U-shape in the predictive ability of the term spread reported by Campbell and Shiller (1991) and others. It might also account for some of the very unusual results obtained when very long-term rates are used. Moreover, because an analogous procedure is used to test uncovered interest parity [e.g., McCallum (1994) and Frankel and Froot (1987)], the bias noted here could also account for some of some the unusual results in foreign exchange rate literature [e.g., McCallum (1994)].

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Table 1: Results from the Conventional Test of the Expectations Theory: September 23, 1974 to October 2, 1997				
	Estimates			
Sample	β	Adj. R^2	Test β = 0	Test β = 1
9/23/74-10/2/97				
All days	0.5415	0.2253	5.20*	4.40*
Settlement Wed.	0.6778	0.3343	14.45*	6.87*
9/23/74-10/5/79				
All days	0.2347	0.0605	0.80	2.62*
Settlement Wed.	0.3653	0.1436	2.05*	3.56*
10/9/79-10/6/82				
All days	1.1681	0.4169	4.74*	0.68
Settlement Wed.	1.0598	0.4518	9.37*	0.53
10/7/82-8/10/87				
All days	0.7793	0.4878	7.13*	2.02*
Settlement Wed.	0.8362	0.7834	15.65*	3.07*
8/11/87-10/2/97				
All days	0.3759	0.2281	3.32*	5.51*
Settlement Wed.	0.7374	0.5787	15.22*	5.42*

*indicates statistical significance at the 5 percent level.

Table 2: The Conventional Test and the Volatility of the Federal Funds Rate

	β	Adj R^2
all days (5,805)	0.5415	0.2253
LSD (338)	0.8804	0.4804
not LSD (5467)	0.4262	0.1396
LSD and SW (120)	0.9167	0.5757
SW but not LSD (725)	0.3940	0.1089

[1/]The number days in each sample period is in parentheses.

Table 3: The Size and Power of the Conventional Test of the Expectations Theory			
	Experiment 1	Experiment 2	Experiment 3
$\hat{\beta}$	1.273	.861	.611
Range $\hat{\beta}$	.038 – 2.421	-.065 – 1.343	-.029 – 1.139
S.D.	.335	.166	.192
$\overline{R}^2$	.485	.536	.497
Prob. reject $[\beta = 1]^{1/}$	.406	.390	.859
Prob. reject $[\beta = 0]^{1/}$	.971	.933	.959

These results are based on 5,000 estimates of Equation 5 using data generated from each of the following DGPs. In each experiment the initial value of the rate was set at its long-run equilibrium level and the DGP generated 200 sample outcomes before drawing the 450 observations used to estimate the equation. The horizon was 91 periods.

DGP 1

$$R_t^m = .0980 + .9851 R_{t-1}^m + .3690\gamma_t$$

$$R_t^n = 3.0199 + .5415 R_{t-1}^m \;^{2/}$$

DGP 2

$$R_t^n = .0033 + .9993 R_{t-1}^n + .0629\varepsilon_t$$

$$R_t^m = R_t^n + \theta_t$$

$$\theta_t = .0505 + .5740\theta_{t-1} - .0622\theta_{t-2} + .1246\theta_{t-3} + .0591\theta_{t-9} + .08858\theta_{t-10} + .1202\theta_{t-19} + .3256\eta_t$$

DGP 3

$$R_t^n = .0033 + .9993 R_{t-1}^n + .0629\varepsilon_t$$

$$R_t^m = .0980 + .9851 R_{t-1}^m + .3690\gamma_t$$

[1/] The standard errors were adjusted without the Newey-West correction [see Hamilton (1994), pp. 280-82]. In some replications the estimated adjusted variance was negative; hence, these probabilities are based on fewer than 5,000 replications. Specifically, the probabilities are based on 4,660, 4,850, and 4,758 replications of experiments 1, 2, and 3, respectively.

[2/] $R_t^n = \theta_0 + \theta_1 R_{t-1}^m$, where $\theta_0 = (\mu^m / k(1-\psi))[k - (\psi(1-\psi^k))/(1-\psi)]$ and $\theta_1 = (\psi(1-\psi^k))/k(1-\psi)$.

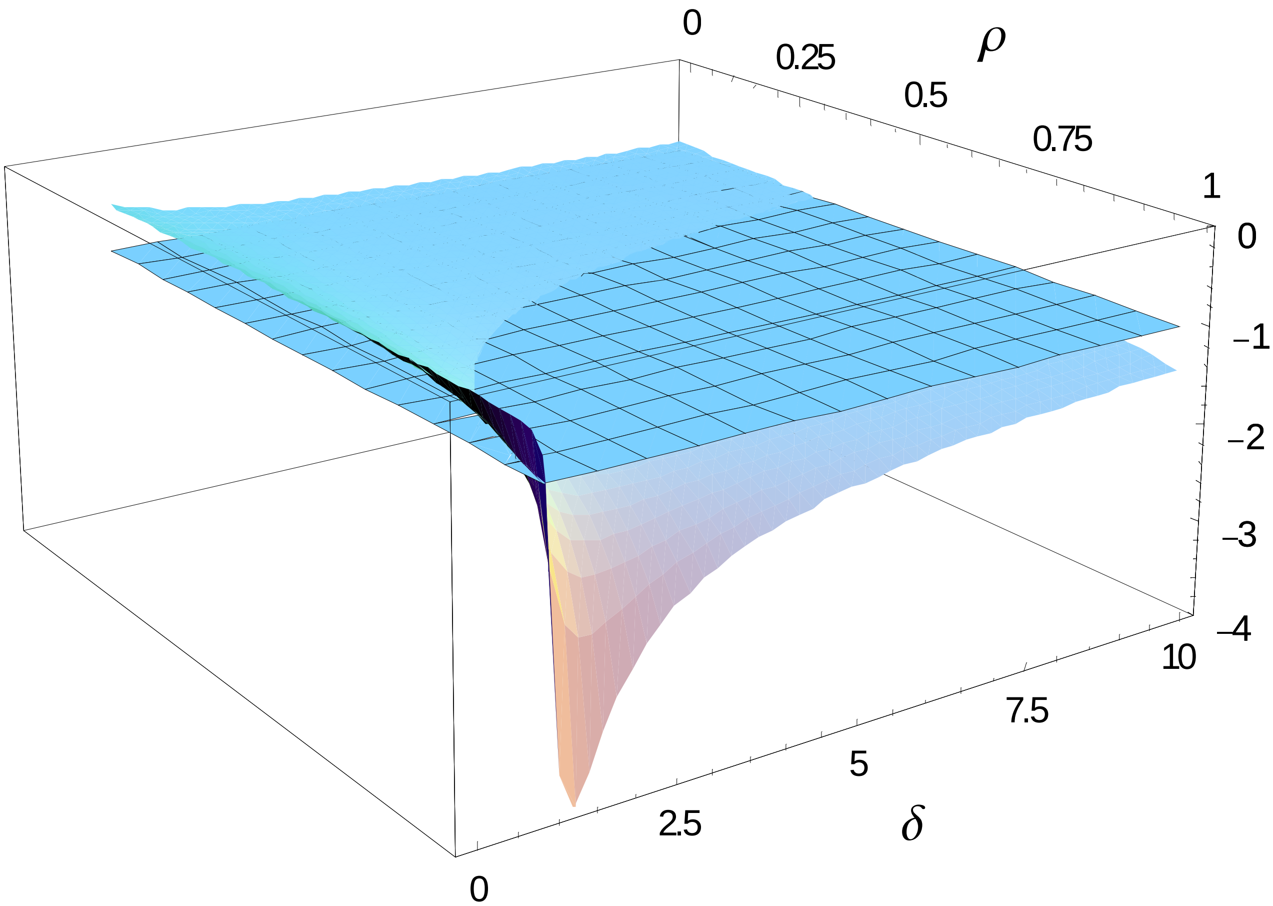

Figure 1: The Bias Factor for $\delta \in [0,10]$ **and** $\rho \in [0.01, 0.99]$

Figure 2: The Independent and Dependent Variables of Equation 5
(September 23, 1974 - October 2, 1997)

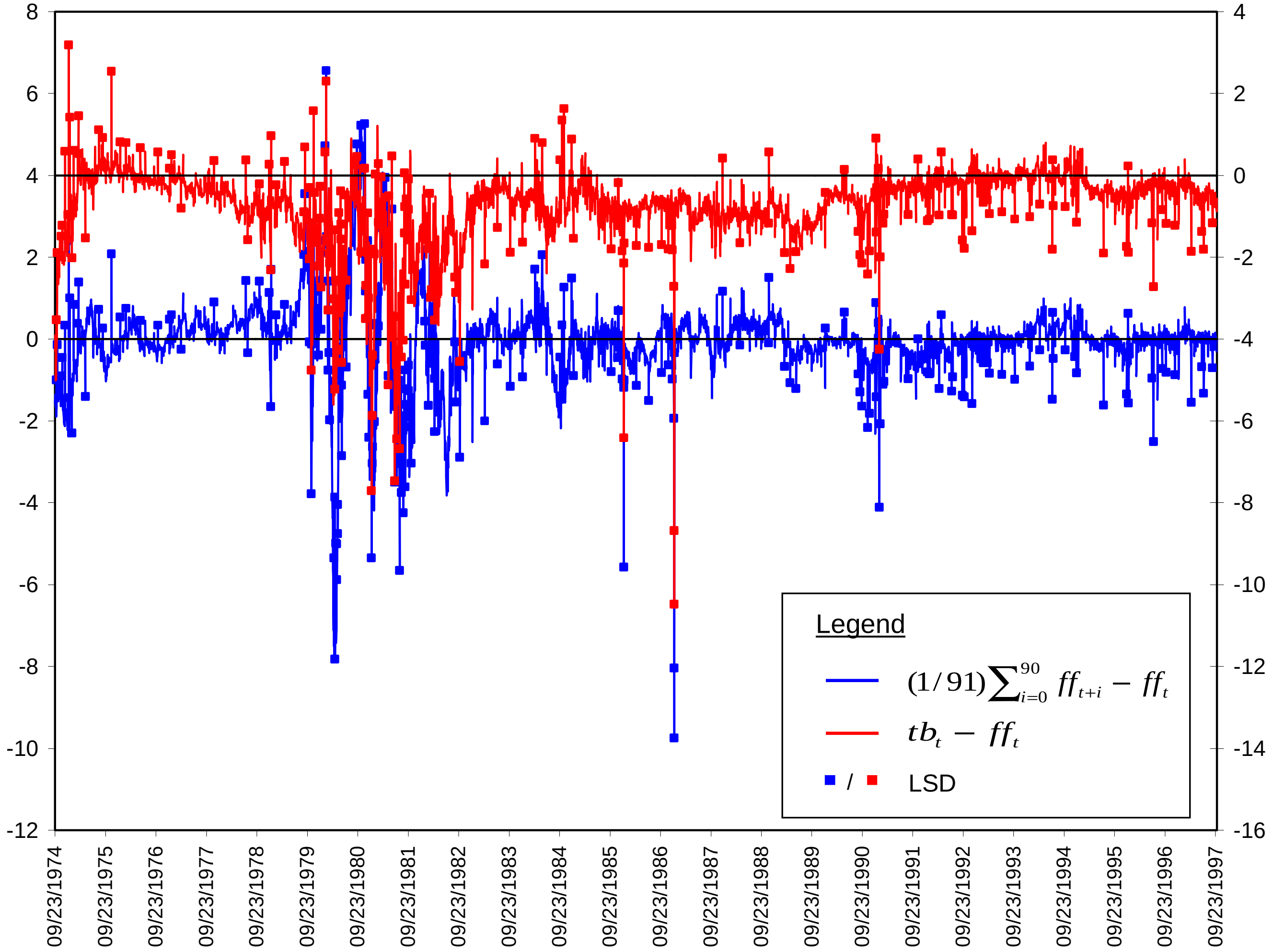

Figure 3: The Independent and Dependent Variables of Equation 5 and the T-Bill and Federal Funds Rates (October 9, 1979 - October 6, 1982)