

## TRACKING ERROR AND VALUE AT RISK

### Summary:

Statistical measures are being used with greater frequency to manage the risk of fixed income portfolios. Two of the measures used are tracking error and value at risk. Both measures rely on analysis of the variability of the market value of the securities held in the portfolio and the relationships between the changes in value of these securities. Statistical measures such as standard deviation and correlation are used to simplify the calculations. In some cases these approximations may lead to erroneous results.

### Tracking error

Tracking error can be defined as the difference in performance of a particular fund relative to a benchmark portfolio. For past results, tracking error can be calculated easily by subtracting the total return of the portfolio for a given period (such as one month) from the total return of the benchmark for the same period. The standard deviation of the monthly differences can be calculated to summarize the results.

For a given set of holdings, the future tracking error of the portfolio can be estimated based upon statistical analysis. This analysis is performed by evaluating the range of potential returns on each instrument and the relationship between the returns on the instruments. While these relationships are quite complex, it is possible to perform the necessary calculations in a relatively straightforward manner if certain simplifying assumptions are made. One such assumption is that the returns for individual securities in the portfolio

### Quantitative Analysis

Agency	Mat	CPN	WAM	AGE	MARKET		SPREAD TO			OPTIONS ANALYSIS			RISK		PREPAYMENT			
					June Settlement		WAL	Zero	Fwr	Option	Fwd Curve	Dur	Convex	Base	FWD	Base	FWD	
					Price	Yield												TRSY
FNMA	30	6.0	343	17	91-29	7.37	70	70	70	60	10	0	5.5	0.2	113	113	9.91	9.9
FNMA	30	6.5	354	6	94-19	7.39	72	71	71	52	19	0	5.3	-0.1	114	114	10.54	10.5
FNMA	30	7.0	354	6	97-4	7.51	83	83	82	51	30	1	4.8	-0.5	120	114	10.36	10.7
FNMA	30	7.5	353	7	99-14	7.64	102	101	97	50	47	3	4.1	-0.9	164	128	8.54	10.0
FNMA	30	8.0	352	8	101-15	7.72	114	112	113	46	67	0	3.3	-1.2	217	183	6.99	7.9
FNMA	30	8.5	350	10	103-7	7.54	104	100	113	37	76	-12	2.3	-1.2	365	272	4.41	5.7
FNMA	30	9.0	345	15	104-25	7.26	83	68	83	22	61	-14	1.7	-0.8	438	376	3.51	4.1
FNMA	30	9.0	336	24	107-7	7.03	57	48	65	13	52	-16	1.8	-0.5	382	339	3.77	4.3
FNMA	30	10.0	294	66	109-2	6.91	45	44	62	25	37	-17	1.9	-0.2	375	340	3.75	4.1

Based on Andrew Davidson & Co., Inc. OAS model and prepayment model

reflect a joint normal distribution. This implies that if you know the means, standard deviations and correlations of the returns of each instrument you can calculate the mean and standard deviation of the portfolio as well as its tracking error relative to a benchmark portfolio.

If the tracking error is expressed as a standard deviation, and the tracking error reflects the assumed normal distribution, then the portfolio return should not differ from the benchmark return by more than this amount approximately two-thirds of the time. In this way, tracking error provides a measure of how much the portfolio returns may differ from the benchmark returns.

### **Value at Risk**

Value at Risk (VaR) addresses a similar but different question. VaR represents the amount which a portfolio may decline in value over a given time period at a specified degree of confidence. For example, VaR could indicate the amount of money a portfolio might lose over a one week time period one percent of the time. Value at risk is frequently used to help determine the amount of capital required to support a given portfolio. If the VaR for a reasonable time horizon at a small enough probability level is less than the amount of capital, then it is unlikely that adverse changes in the value of the portfolio would wipe out the capital.

As with tracking error, VaR is more easily calculated if the changes in value of the securities in the portfolio are assumed to be jointly normal. Using this assumption the value at risk can be computed by first computing the portfolio standard deviation and then calculating the VaR.

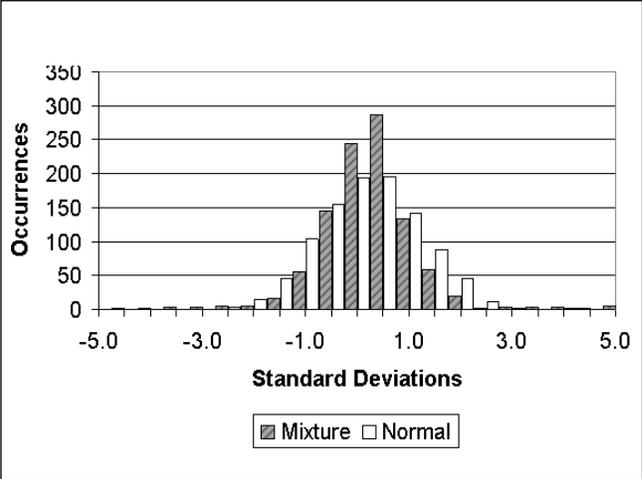
Unfortunately returns do not always reflect normal distributions. When returns differ from normality, risk measures developed based on that assumption may be misleading. The degree of error will depend on the extent to which the portfolio returns differ from the normal distribution assumption as well as the nature of the risk measure.

### **Fat tails**

To demonstrate the potential error, we constructed a distribution that is not normal. The distribution has the feature that extreme events are more likely than in a normal distribution. This assumption corresponds with the observed behavior of many markets. The sample distribution was constructed using a method called "mixture distributions." In essence the new distribution represents a combination of two normal distributions, one with a low standard deviation and one with a high standard deviation. In most cases returns are generated by the low standard deviation component. However in rare cases the returns come from the riskier distribution. For additional information, see "Value at risk for a mixture of normal distributions: The use of quasi-Bayesian estimation techniques," by Subu Venkataraman, Economic Perspectives, Federal Reserve Bank of Chicago, March/April 1997.

We also created a normal distribution that has the identical standard deviation as the mixture distribution. 1000 sample returns were created using each distribution. Figure 1 (see next page) shows the results from each distribution. The mixture distribution is "fat tailed." It has a taller peak in the middle and has a wider spread of outcomes.

Figure 1



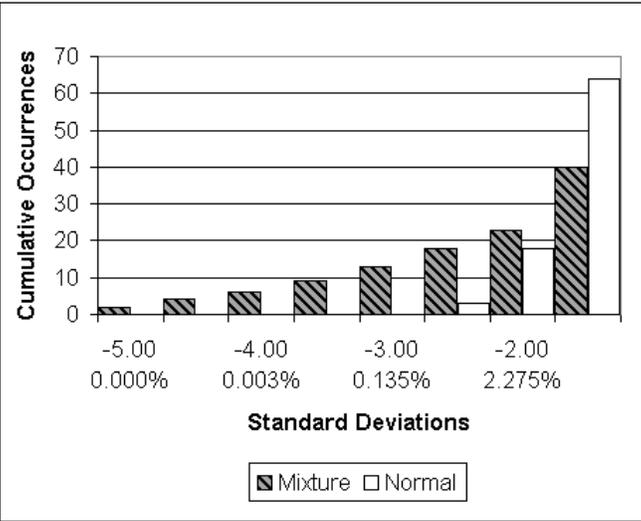
**Consequences**

If we use the statistical measures derived based upon standard deviation we may come to incorrect conclusions when the distribution is not normal. The effects may be more or less severe depending on the type of analysis we are performing. For tracking error analysis we wanted to make sure that the portfolio returns did not differ from the benchmark by too great an extent. In a normal distribution, 2/3 of the results are within one standard deviation of the mean. For our mixture distribution, it turns out that over 80% of the results were within one standard deviation of the mean. Thus the portfolio may be seen as more conservative than the measure would indicate. Of course the range of results outside of one standard deviation is wider than for the normal, so there is some additional risk that standard deviation is not capturing.

For the VaR measure the situation is reversed. Figure 2 shows the cumulative number of results at various standard deviations. The theoretical probabilities are shown on the bottom row of the x-axis below the number of standard deviations. Based upon the standard normal distribution there is a small probability of a result of greater than three standard deviations. Less

than 2 in 1000 times should a loss of that size occur. In this particular case, we did not get any results of that magnitude from the normal distribution. However for the mixture distribution there were 13 occurrences beyond the three standard deviation mark. This is almost 10 times the theoretical result for a normal distribution. If the VaR is used to determine the confidence that the position can withstand a large move in the portfolio value then our conclusions could be quite misleading. To get to the same degree of confidence for the mixture distribution we would need about double the protection or six standard deviations to reduce our risk below the 2 in 1000 level.

Figure 2



**Conclusion**

Numerical measures of risk are giving us better insight into the performance of our portfolios. In constructing these measures we face tradeoffs between the complexity of the calculations and the reliability of the results. Often we assume that returns are normal. Generally this is a safe assumption because many random series closely resemble a

normal distribution. However, in some cases return patterns may not be normal. In these cases we need to explore the impact of our decision to simplify and determine if the analysis requires a more complex

method. These considerations are especially important for measures such as VaR, where the consequences of exceeding a risk target are great.

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