CSIM
Basic Report

Value-at-Risk in portfolio management

CSIM – CREDIT SUISSE INVESTMENT MODELS
Dr. Patrick Gügi
Dr. Günter A. Hobein
Dr. Martin Schlatter
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CSPB PTMQ
P.O. Box 500
CH - 8070 Zurich

Dr. Patrick Gügi, Director

Editor: Dr. Patrick Gügi

Contributors to this issue:
Dr. Günter A. Hobein
Dr. Martin Schlatter

Layout
Synia Kaross

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Summary

Value-at-Risk (VaR) is a risk measure that enables you to determine how much the cash value of a portfolio could decline over a defined time horizon with a given probability as a result of adverse changes in market conditions.

This Basic Report presents the principles of Value-at-Risk and the concept's applications. Using simple examples and approaches, this Basic Report explains how you can implement the concept of Value-at-Risk to your advantage.

VaR allows you to explicitly define your portfolio hedging needs by means of two parameters - confidence level and time horizon. Of course, other parameters exist, as the expected return on the portfolio and the expected risk of the portfolio.

The usual three or four VaR methodologies dealt with in the body of literature on the subject are more or less similar in terms of quality, but are vastly different in terms of the amount of resources they entail in calculating VaR (and are thus vastly different in terms of practicality).

In 1996, the Delta-Normal method of calculating VaR was implemented in the CREDIT SUISSE INVESTMENT MODELS (CSIM), which are available to all portfolio managers employed by CSPB. Delta-Normal is the most suitable VaR methodology for portfolio management.

The first part of this Basic Report provides you with background on VaR. Afterwards, various techniques for calculating VaR are introduced, including a detailed description of the Delta-Normal methodology. Part three presents concrete examples of how VaR can be employed using CSIM. CSIM enable any portfolio manager to calculate the VaR for any asset portfolio managed by Credit Suisse. A few critical observations round out the report.

INTRODUCTION

Risk is the danger of not achieving a certain return. Most of the risk measures used in modern financial-market and portfolio theory assume that returns are randomly distributed. The statistical distribution of returns on the Swiss equity market from 1926 to 1998 is shown in Figure 2 on page 4.

On the basis of the return distribution, modern portfolio theory employs various measures in its attempt to describe risk as a single numerical figure. Such risk measures include volatility, variance, semivariance, downside risk, and failure probability.[1]

In order to quantify risk, it is therefore necessary to forecast the distribution of future returns and to select a suitable measure of risk. Value-at-Risk (VaR) is a new risk measure. This Basic Report describes the various methodologies for calculating VaR and explains how the concept is implemented in CREDIT SUISSE INVESTMENT MODELS (CSIM).[2]

CSIM provide Credit Suisse Group investment specialists with quantitative tools for all of the steps in the investment decision-making process. CSIM enable approximately 500 users in Credit Suisse Group's investment units to efficiently employ the latest advances in financial-market and portfolio theory in formulating strategies for each portfolio.

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[1] See Gügi, p. 28 et seq., for a description of the various concepts.
[2] See Gügi/Schmid (1996) for detailed information on CSIM.

In the process, quantitative analysis of the risk-return profile of each portfolio stands in the foreground. The composition of each portfolio is optimized taking into account current market data and the specific wishes of each client. In addition, CSIM offer tools for measuring performance, analyzing the portfolio, and graphically illustrating the two (including performance attribution).

WHAT IS VALUE-AT-RISK?

VaR expresses in currency units the expected maximum loss that may be incurred over a defined time horizon and within a specified confidence interval.

Example 1 Your bank informs you that the VaR for your trading portfolio over a one-day holding period is CHF 35 million at a confidence level of 99%. What does this mean?

1) Under normal market conditions, in 99 out of 100 cases, or on 99 out of 100 trading days, or with a 99% probability, a single day's loss on the portfolio will not exceed CHF 35 million.

Or in other words...

2) Under normal market conditions, in 1 out of 100 cases, or on 1 out of 100 trading days, or with a 1% probability, a single day's loss on the portfolio will exceed CHF 35 million.

Return and volatility

Figure 1 depicts the historical development of the Swiss equity market.[3] On a long-term average, the Swiss stock market has generated an annual return in excess of 10%. Risk, in this case, is the danger of failing to achieve an average annual return of 10%. The literature on risk theory employs volatility and standard deviation as measures to determine the variance about the 10% mean annual return. The volatility figure for the Swiss equity market is approximately 20% for the given period.

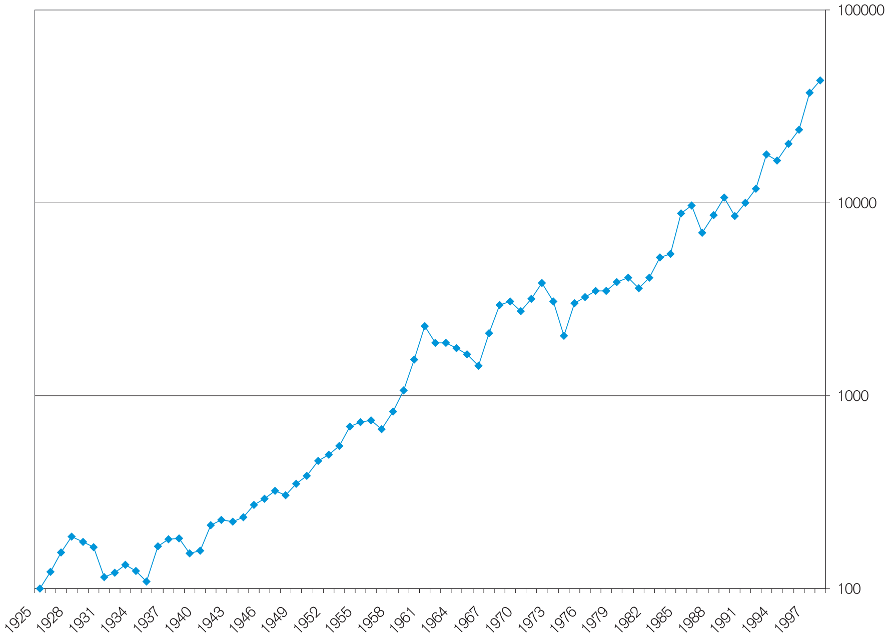

Figure 1: Historical return on the Swiss equity market from Dec. 1925 - Dec. 1998.

[3] According to the Pictet-Rätzer equity index, the Swiss stock market generated an average annual return of r = 10.56% form 1926 to 1998, while its risk value was σ = 20.65%

Normal distribution of returns

Risk measures attempt to assess - in a single numerical figure - the probability of not obtaining an expected return. In order to do so, it is necessary to know the distribution of the potential expected returns. Figure 2 displays the return distribution for the Swiss equity market from 1926 to 1998. The illustration shows that the actual distribution very closely resembles the normal distribution plotted in the chart. If the returns are normally distributed, the computational advantages of the normal distribution, as opposed to other types of distributions, can be exploited. The shape of the distribution of normally distributed returns is determined by the expected return (r) and the volatility (σ).

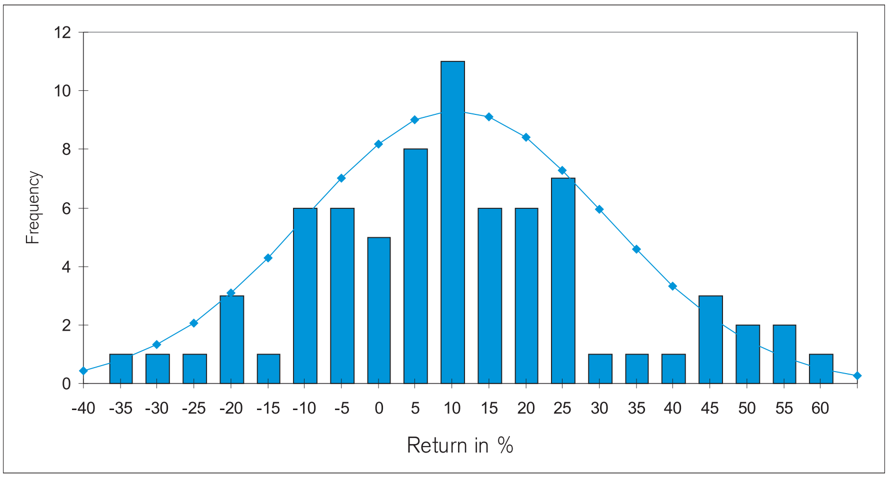

Figure 2: Normal distribution compared with actual distribution of Swiss stock-market returns from 1926 to 1998.

Thus, for each normal distribution with an expected value r,

68% of all possible values lie between $r - \sigma$ and $r + \sigma$, and

95% of all possible values lie within the interval $(r - 2 \cdot \sigma, r + 2 \cdot \sigma)$.

Applied to our example looking back over numerous years of Swiss equity returns, a normal distribution implies that next year there is

a 68% probability that the return on the Swiss stock market will be between -10% (= 10% - 20%) and 30% (= 10% + 20%), and

a 95% probability that the return will be between -30% (= 10% - 2 · 20%) and 50% (= 10% + 2 · 20%).

What do the values realized by the Swiss equity market indicate? In order to find out, we count the number of returns situated in the 95% interval in Figure 2:

There is 1 return to the left of -30% and there are 3 returns to the right of +50%, which adds up to 4 out of total of 73 yearly observations. Four is 5.5% of 73. This value is slightly above the 5% figure that one would expect from a normal distribution.

In reality, returns on securities do not precisely fulfill the normal-distribution assumption. Studies[4] show that extreme returns, in particular, occur more frequently than normal distributions suggest. When putting risk measures into practice, the key is to estimate the future return distribution with as little error as possible. However, since it is very difficult to

[4] For more detailed information, readers should refer to specialized literature on the subject. For example, see Gügi, p. 32 - 40.

forecast deviations from the normal distribution (extreme returns, for example), especially for mixed portfolios, the normal-distribution assumption is the most sensible solution. A normal distribution should not be assumed when, in a given portfolio, the proportion of derivatives with non-linear payoffs (options) exceeds a certain value.

The confidence level

When, as in the above example, the deviation from the expected value is observed in both directions (that is, above and below the expected value), one speaks of a two-sided confidence interval. For an investor, however, the possibility of making a very large profit is not a risk. Only negative deviations from the expected value constitute risk for an investor. Hence, only one-sided confidence intervals are relevant.

The level of security required by the investor is reflected in the confidence level chosen. The more security an investor requires, the higher the confidence level. The impact that the selection of a particular confidence level has on the potential loss is illustrated in Figure 3 below.

The most commonly selected confidence levels are 90%, 95%, and 99%.[5] The significance of these three levels with regard to our example using the historical return on Swiss equities is shown below.

90%: Return > -16.0%	95%: Return > -23.5%	99%: Return > -37.5%

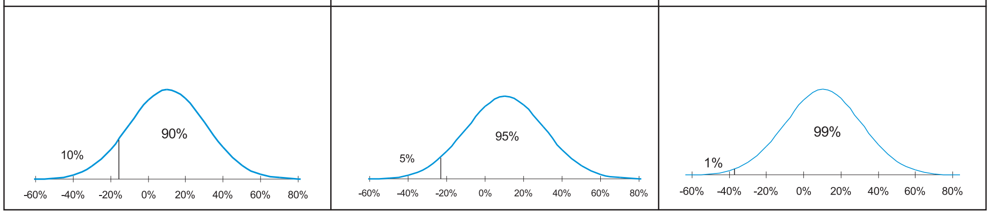

Figure 3: The most common confidence levels viewed in Swiss equity example (1926 - 1998: r = 10.56%, σ = 20.65%).

Example 2 What is the expected maximum amount that an equity portfolio worth 1,000,000 can lose at a confidence level of 95%?

The downside potential for the coming year amounts to 235,000.

Why?

Because according to the middle graph, there is a 95% probability that the portfolio will realize a return greater than -23.5% over the coming 12 months.

Hence, the downside potential at the 95% confidence level is:
1,000,000 x -23.5% = -235,000.

[5] The Basel Committee on Banking Supervision advises Swiss banks to calculate daily VaR at the 99% confidence level over a 10-day holding period.

Graphic and mathematical representation of Value-at-Risk

If the returns are normally distributed, one times and two times the volatility is added and subtracted from the mean return. This generates single and double fluctuation ranges, which are formulated here on the basis of the historical return on the Swiss equity market.

Table 3	Month	3	6	12
	Single	101 … 121	109 … 137	133 … 172
	Double	91 … 131	96 … 151	113 … 192

Plotted on a graph, the fluctuation ranges appear as follows:

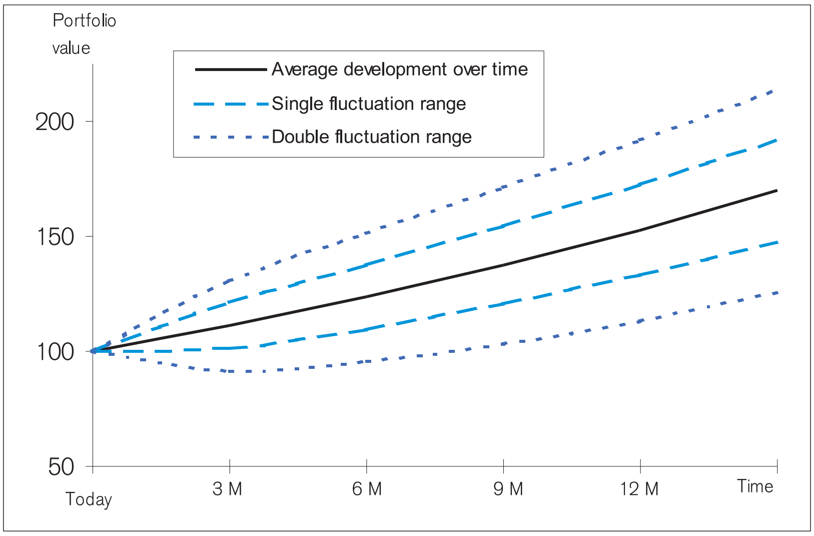

Figure 4: Single and double fluctuation ranges surrounding expected portfolio value over time.

Figure 5 shows what Figure 4 looks like when the normal distribution curves for 3 and 12 months are added.

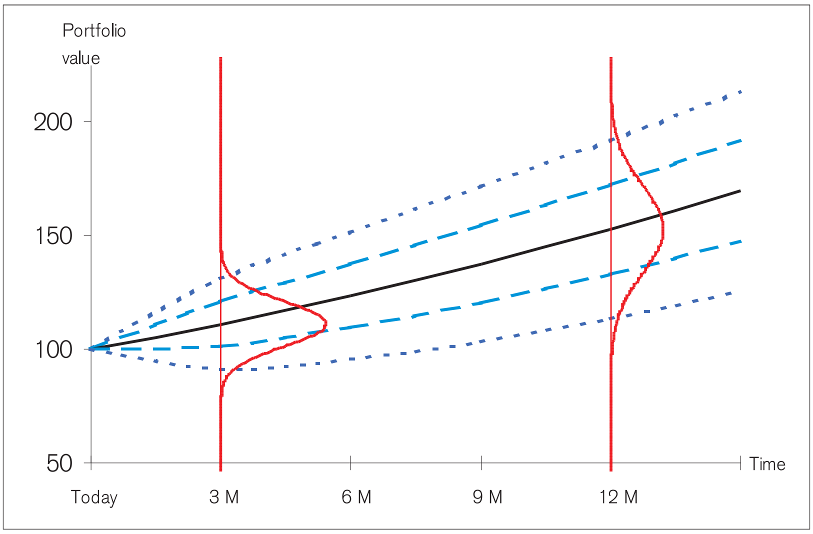

Figure 5: Single and double fluctuation ranges surrounding expected portfolio value over time.

Figure 5 can be interpreted as follows:

1. With a 95% probability, or in 95 out of 100 cases, the portfolio (PF) value lies between the outer, dark-blue curves (short dashes).

2. With a 68% probability, or in 68 out of 100 cases, the PF value lies between the inner, light-blue curves (long dashes).

Moreover, the value at risk can now be visualized.

The black curve in the middle describes the continual development of a portfolio with a constant return. Anything below this curve is undesirable. The blue perpendicular segments in Figure 6 represent the downside potential over 3 and 12 months at a 97.5% confidence level.

This corresponds precisely to the definition of Value-at-Risk (VaR).

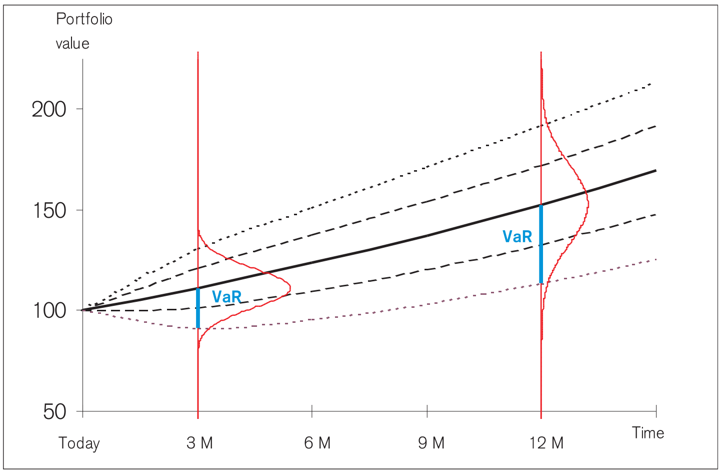

Figure 6: Graphic representation of VaR over time.

We can draw the following mathematical conclusions:

1. Table 3 indicates that the double fluctuation range (+/- 2 standard deviations from the mean) stretches from 91 to 131 on a 3-month horizon, while the expected portfolio value is 111.

2. The double fluctuation range is identical to a two-sided 95% probability, or confidence interval.

3. Therefore, the VaR for a pure CH equity portfolio at the 97.5% confidence level over a 3-month holding period is 20 currency units (111 - 91 = 20).

4. On a 12-month horizon, the VaR is 40 (153 - 113 = 40).

When we generalize the conclusions drawn from the charts, we arrive at a preliminary, general method of calculating VaR.

VaR = the expected return minus the lower "loss threshold"

Expressed mathematically:

$$VaR = W_0 \cdot (1 + r)^{\Delta t} - W_0 \cdot z_\alpha \cdot \sigma \cdot (\Delta t)^{0.5}$$

where:

- Δt is the time horizon (in months);
- W_0 is the current portfolio value;
- r is the expected return (per month);
- σ is the expected volatility (per month);
- z_α is the normal distribution value (confidence level).

If the expected return is ignored or set at 0% over a very short time horizon, one speaks of absolute VaR. In most cases, the expected return is taken into account, which is why the literature in the field refers to the concept of "VaR relative to the mean". Another option is to calculate VaR relative to a benchmark. Here the term "Benchmark-Value-at-Risk", or "BVaR". is commonly used.

Forecasting portfolio volatility

The previous explanations refer to a portfolio consisting of a single investment. The inclusion of a bond denominated in a foreign currency complicates the method used to determine volatility.

Example 3	An investor holds CHF 100 million in US Treasury bonds. How great is the VaR for a 12-month holding period at a 95% confidence level?
Risk exposure:	The bonds are subject to interest risk and CHF/USD exchange-rate risk.
Volatility:	The volatility of the Treasury yield is 4%, and the volatility of the CHF/USD exchange rate is 10%.
Portfolio volatility:	The overall volatility is not merely the sum of the individual volatilities. The reciprocal relation (correlation) between the factors that influence the return must also be considered. In this case, the correlation equals -0.1. From the equation $\sigma_{PF}^2 = 0.04^2 + 0.1^2 + 2 \cdot (-0.1) \cdot 0.1 \cdot 0.04 = 0.011$ we derive a volatility of $\sigma_{PF} = 10.4\%$
VaR	PF value x (-1.65) x σ_{PF} = -17.2 million[7]

This example combines two different revenues that are both subject to fluctuations: revenue from fluctuating bond yields and revenue from fluctuating exchange rates. The VaR calculation in this example is more complex than the one in example 2 because the correlation must also be taken into account.

Forecasting the volatility of a globally diversified portfolio is considerably more complex. First, one must determine which variables influence the portfolio's volatility. Then, the volatilities of the individual risk factors and their correlations must be estimated, and a covariance matrix must be created out of the covariances and variances. Finally, the individual assets in the portfolio must be broken down into their respective risk factors.[8]

Covariance matrices in CSIM

CSIM base covariance matrices on the most significant indices and subindices. CSIM utilize approximately 50 indices per country, including subindices (e.g., sector indices for equity markets and indices for specific bond maturities). The estimations of the covariance matrices are based on historic time series that are exponentially weighted. With exponential weighting, recent events are assigned a heavier weighting than earlier events.[9]

Starting from the estimated covariance matrix, CSIM calculate VaR using the Delta-Normal methodology described below.

[7] Value at 95% confidence level: $z_{5\%} = -1{,}65\%$
[8] This lengthy process is automated in CSIM. Examples of how to break down investments such as options and futures into their constituent elements can be found in Gügi, p. 165 et seq.
[9] See Burkhard/Gügi/Schlatter, p. 6 et seq., for a detailed explanation of exponential weightings.

Methodologies for calculating VaR

A. Delta-Normal methodology

As we have seen in the preceding sections, knowledge of the return distribution is of central importance in calculating VaR. Delta-Normal methodology assumes that the returns on all investments are normally distributed. Since one of the properties of a normal distribution is that the sum of the normally distributed values is also normally distributed, the distribution of the returns on a given portfolio is likewise normal. In order to measure the portfolio's volatility, a covariance matrix must be estimated in advance.

Premises and prerequisites:

- The individual securities in a portfolio must be broken down into their components and risk factors.
- The expected volatilities of the various risk factors (returns, indices, exchange rates, changes in interest rates) and the correlations between them must be forecast.
- All risk factors are normally distributed.
- Linear payoffs adequately represent price changes for individual components of the portfolio.

Advantages:

- Delta-Normal methodology is based on well-known techniques in modern portfolio theory (Markowitz) and is widely disseminated thanks to RiskMetrics® and CSIM.
- If input values such as the covariance matrix and factor sensitivity are known, the VaR can be calculated in an Excel spreadsheet.

Disadvantages:

- The assumption that all risk factors are normally distributed is rarely true.[10]
- The VaR calculation is only as good as the underlying data sets. CSIM, which automatically break down securities into their respective risk factors, provide you with a fundamental frame-work.
- The linearization of prices is problematic when one is dealing with portfolios with a large proportion of options.

Delta-Normal is the most widely used methodology. Delta-Gamma methodology is recommendable for option-heavy portfolios. This procedure expands on Delta-Normal methodology by also taking the gamma into consideration.[11]

B. Simulations

Another possibility is to disregard the normal-distribution assumption, opting instead to simulate the portfolio's return distribution. For simulation purposes, direct historic data (historical simulation) or a set of computer-generated random variables (Monte Carlo simulation) can be used.

Simulation methods do not rely on an assumed distribution because they apply existing or computer-generated series of price changes to the portfolio in question. Monte Carlo simulation is extremely intensive in terms of computational complexity and the computer resources it requires. Simulation methods are recommendable primarily for option-heavy portfolios.

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[10] However, since forecasting deviations from a normal distribution involves a very large degree of uncertainty, the normal-distribution assumption remains the most practical solution.

[11] For a detailed description of Delta-Gamma methodology, readers should refer to specialist literature, e.g. Rouvinez.

C. Additional methods
In addition to the methods described above, the trading departments of banks often employ other methods. These are largely variations on the aforementioned methods.

Stress testing and worst-case simulation assess what effect concrete scenarios and extreme events have on a portfolio. The portfolio is revalued in each scenario, and the resulting loss is calculated.

One could conceivably also utilize particularly eccentric variances, covariances, or correlations (for example, one could employ only positive correlations or set all correlations at +1).

IMPLEMENTING VALUE-AT-RISK

VaR parameters that clients can set themselves

Two parameters enable the user to directly influence the VaR and alter his or her risk profile:

Confidence level:
The greater the confidence level, the higher the VaR (see Figure 7).

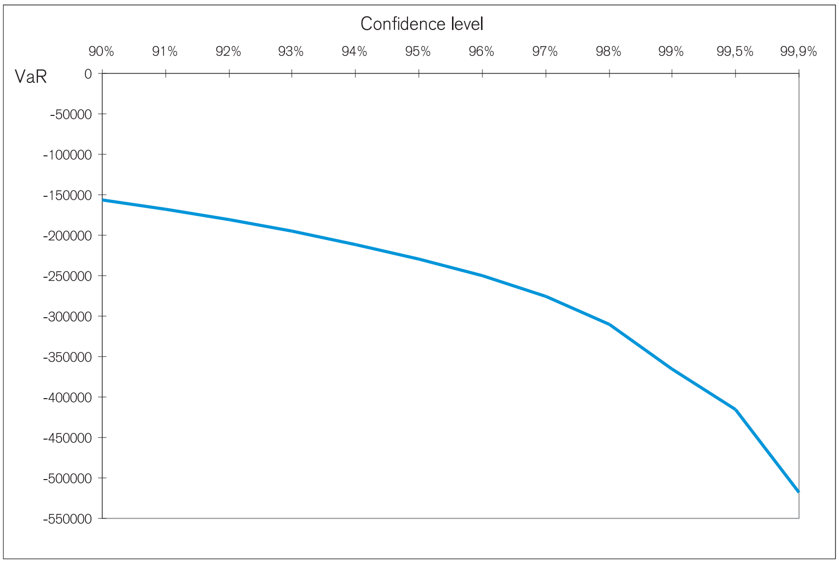


Figure 7: The effect of the confidence level on the VaR (PF value: CHF 1,000,000, r = 10%, σ = 20%, Δt = 12 months).

Time horizon:
The impact of the time horizon on the VaR depends on the sum of the expected return.

I) If the expected return is disregarded, the rule is: the longer the time horizon, the higher the VaR.

II) If expected returns are taken into account (as is normally the case), each individual return must be examined separately (see figures 9 and 10).

The "Parameter VaR" window in CSIM

After CREDIT SUISSE INVESTMENT MODELS (CSIM) have calculated the analysis overview for the loaded portfolio and the benchmark ("Analysis", "Calculate Analysis"), the aforementioned VaR parameters can be altered under "Window" and "VaR Parameter."

1. Time horizon ↔ "Time in Months"
2. Confidence interval ↔ "Confidence Level"

Figure 8: "Parameter VaR" window in CSIM.

The "Return over Timehorizon" drop-down box gives you the option of selecting either "expected" or "0%". This enables the portfolio manager to choose whether the expected return is to be considered over the time horizon ("expected"), or whether it is to be ignored ("0%").

The final drop-down box ("VaR type") determines whether the VaR is calculated relative to the benchmark ("B-VaR") or whether it is calculated in absolute terms ("absolute").

What other factors affect VaR?

Alongside client-selected parameters such as confidence level and time horizon, there are other determinants such as, for example, expected return and volatility. The relation between these two factors and VaR is:

A higher return lowers the VaR.

Higher volatility alone raises the VaR.

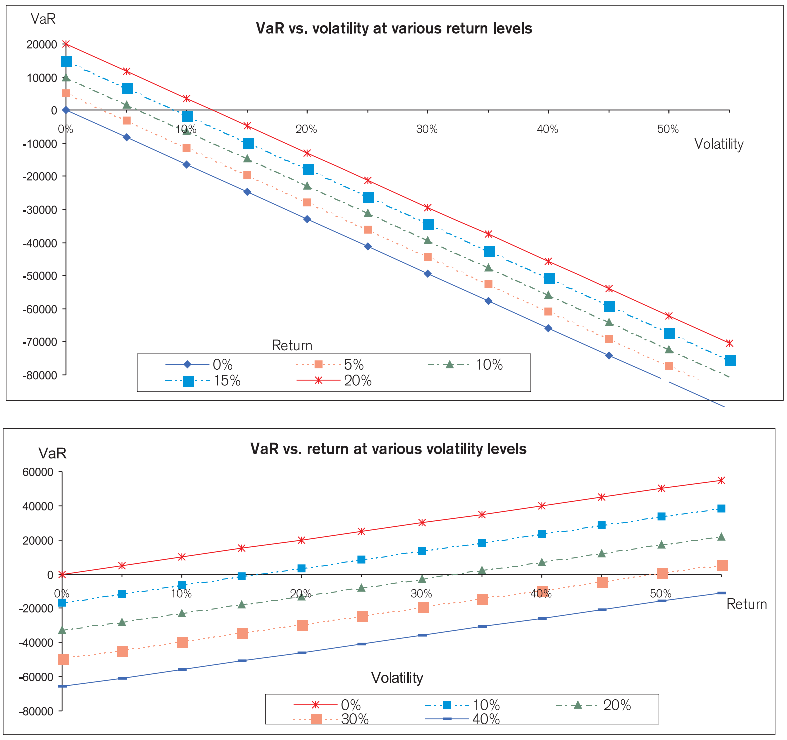
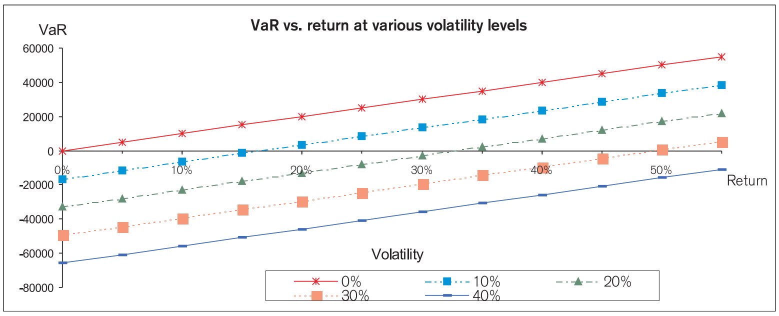
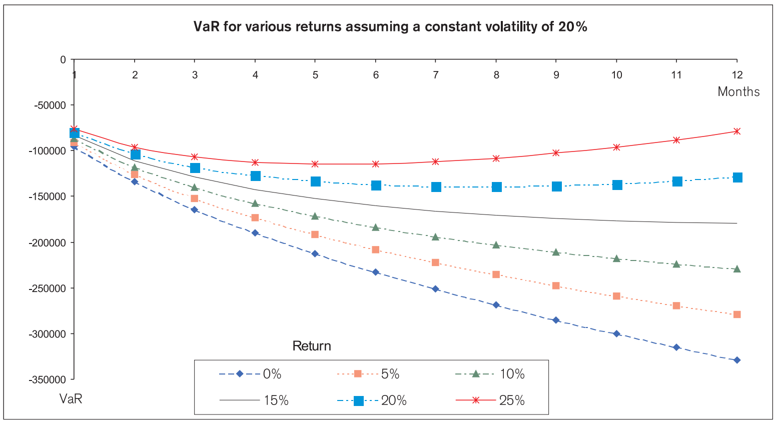


Figure 9: Impact of various returns and volatilities on VaR (PF value: 1,000,000; time horizon: 12 months).

The graphs become more interesting when, as in Figure 10, returns and volatility are observed over time,





Figures 10: Impact of return and volatility on the VaR over time
(PF value: 1,000,000; time horizon: 12 months).

Figure 10 illustrates that taking on greater volatility is only worthwhile if the expected return is correspondingly high.

VaR in CSIM and its informative value

Is VaR a useful means of estimating market risk? If yes, how can you incorporate it into your client consultations?

On the basis of the portfolio depicted in Figure 11, we will attempt to assess whether VaR would have covered the extent of the losses recorded in August and September 1998 and would thus have been a reliable risk measure.

Asset Allocation Übersicht

	CHF	EUR	DEM	NLG	FRF	ESP	ITL	GBP	REUR	USD	JPY	ASIA	REST	Total
Geldmärkte	5.6	4.2						0.0		2.4	0.0			12.2
Obligationen														
Wandelanleihen														
Aktien	39.3		2.9	2.4	9.6	5.0	4.2	1.9		19.3	3.1			87.8
Rohwaren														
Andere														
Total	44.9	4.2	2.9	2.4	9.6	5.0	4.2	1.9		21.7	3.1			100.0

Figure 11: Portfolio composition as visualized in CSIM.

We observed and analyzed the evolution of the portfolio value over four months running from 31 July 1998 to 30 November 1998. The portfolio had an initial value of CHF 1,864,741 at the end of July. In order to forecast the maximum expected loss during this period (within a specific probability), we calculated the absolute VaR at both the 95% and 99% confidence levels. The expected return is disregarded in both examples. The time horizon is four months. On the basis of these parameters, the analysis overview on 31 July 1998 produced the following picture.

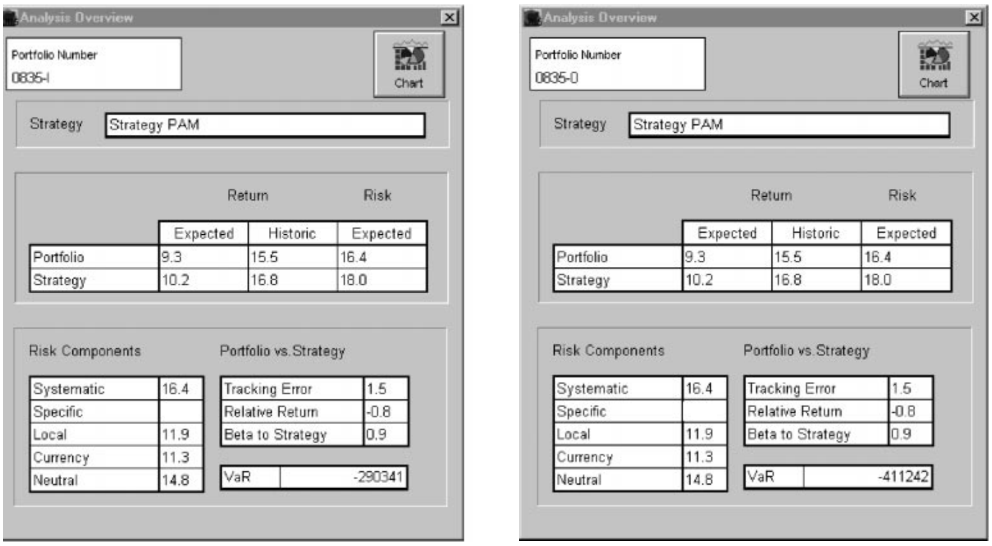


Figure 12: Analysis Overview window in CSIM with a confidence level of 95% (left) and 99% (right).

On 31 July 1998, the portfolio's 4-month 95% VaR was CHF 290,341, or 15.6% of its initial value. This means the client can assume that in 95 out of 100 cases, his loss will be less than CHF 290,341, or the return on his portfolio will be greater than -16%.

At the 99% confidence level, the client can expect to lose no more than CHF 411,242, or 22% of his portfolio's initial value, in 99 out of 100 cases.

The graph below tracks the change in the value of the portfolio over the time horizon defined above (the portfolio value on 31 July 1998 is indexed to 100).

It is clearly evident that the losses recorded in August and September were less than the 99% VaR, which means that the maximum expected loss (at the 99% confidence level) was not exceeded. Hence, the VaR concept proves to be a useful risk measure, even in turbulent times.

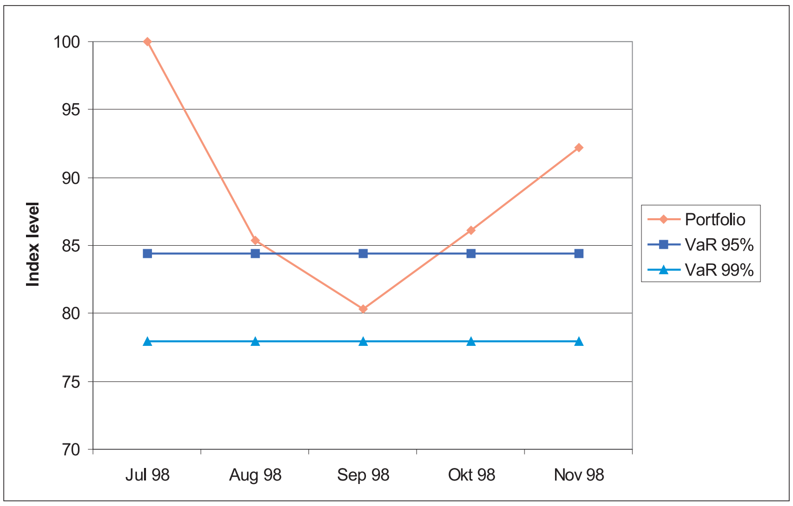


Figure 13: Portfolio value over time, and 95% and 99% confidence levels.

But what about the 95% VaR? The portfolio's slump is greater than the VaR figure. Does this result contradict the previous claim and raise doubt about the concept's reliability? The answer to this question is a clear "no". A 95% confidence level means there is a 5%

probability that the maximum loss may exceed the computed VaR. The Value-at-Risk concept provides no 100% guarantee that the loss will never be greater than the computed VaR.

Our example demonstrates that there is no single ideal measure of risk. When managing risk, it is advisable to utilize a set of precisely defined risk measures, which invariably should include VaR.

KEEP IN MIND

When dealing with VaR measures, the following considerations should be kept in mind:

- Various approaches and parameters can be used to calculate VaR. Consequently, several different VaR figures may result for a single portfolio. Larger VaR figures are not a priori of poorer quality.
- The oft-used normal-distribution assumption tends to lead to an underestimation of VaR. Historical data show that extreme returns occur with greater frequency than one would expect from a normal distribution.
- The most common approach - Delta-Normal methodology - presupposes a linear return trend, a trend that portfolios with a very heavy proportion of options do not exhibit. More complex approaches such as Delta-Gamma methodology, as well as scenario and simulation techniques, address this problem.
- VaR calculation is only as good as the underlying data set. For example, are the historical time series used for the covariance estimates equally or, better, exponentially weighted, as in CSIM? Do the individual securities in the portfolio have to be broken down into their components manually or, better, is the process automated, as it is in CSIM?
- The VaR calculation is based on forecasted parameters, and must therefore also be construed as a forecast that entails all of the problems inherent to empirical methods of estimation.
- As Figure 13 illustrates, VaR says nothing about the magnitude of the loss that can be incurred beyond the specified confidence interval (100% probability minus the level). Additional risk measures are required to answer this question.
- VaR is a risk measure that estimates market risk. VaR does not address factors such as liquidity risk and implementation risk.
- VaR is only one of many risk measures. If, for example, the calculated VaR is too high, we know that we should lower the portfolio's risk, but we do not know how to reduce it. This task requires other measures that are available in CSIM, such as the Consistency Ratio. [12]

[12] See Gügi/Jaecklin for a description of the consistency ratio.

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